OPERATOR MEANS, MONOTONICITY AND FREE FUNCTION THEORY

MIKLÓS PÁLFIA

Recent advances in the axiomatic theory of operator means of probability measures [3] and non-commutative - also called free - function theory [1] lead us to investigate possible extensions of Loewner's classical theorem on operator monotone functions of a real variable going back to 1934. We provide characterizations of operator monotone and concave functions in several operator variables using LMIs and the theory of matrix convex sets. This completes the work of Agler-McCarthy-Young [1] providing characterizations restricted to commuting tuples of matrices, the case of C^1 real functions of several real variables.

The approach [2] leads to the extension of Loewner's classical representation theorem of operator concave and operator monotone functions from 1934, into the non-commutative several variable situation. Our theorem states that a free function defined on a k-variable free self-adjoint domain is operator monotone if and only if it has a free analytic continuation to the upper operator poly-halfspace $\Pi^k := \{X \in \mathcal{B}(E)^k : \Im X_i > 0, 1 \leq i \leq k\}$ for any Hilbert space E, mapping Π^k to Π.

References

[1] J. Agler, J. E. McCarthy and N. Young, Operator monotone functions and Löwner functions of several variables, Ann. of Math., 176:3 (2012), pp. 1783–1826.

[2] M. Pálfia, Loewner's theorem in several variables, preprint (2016), http://arxiv.org/abs/1405.5076, 47 pages.

[3] M. Pálfia, Operator means of probability measures and generalized Karcher equations, Adv. Math. 289 (2016), pp. 951-1007.

Department of Mathematics, Sungkyunkwan University, Republic of Korea & Hungarian Academy of Sciences, Hungary

E-mail address: `palfia.miklos@aut.bme.hu`

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